

Collatz Series Profile

Background and purpose

This document has been converted from a Mathematica notebook, and partially edited.

The purpose of this notebook is to document key statistical elements in Collatz sequence. In *New Perspectives on the Collatz Sequence.pdf* I have demonstrated that Collatz sequence are composite and that they trace segments of multiple Minimum Track Sequences (cMTS). The cMTS are reverse sequences, and they constitute the main structural elements of the Reverse Collatz Structure (RCS). Background information and summaries of critical concepts used in this notebook can be found in the file, *Collatz Sequences and the RCS.pdf*. The complete presentation of these ideas can be found in *New Perspectives on the Collatz Sequence.pdf* which can be downloaded from my website, BandInOne.com.

Organization of this document

The following section, Methods and Findings, include notes on the methods and data structure, as well as a summary (compilation of multiple runs) and analysis of the findings. The Data section that follows contains the Mathematica input and output. This is followed by the Function section which contains notes on the functions and coding.

Methods and Findings

Method and Data structure

The focus of this report is the cMTS components of Collatz sequences. In the Data section below you will find six different tests with the following parameters. Each test consists of a random sample of 10 000 odd numbers, and each selects from a characteristic range. The ranges are:

- T 1: 2000 to 10^6
- T 2: 10^6 to 10^7
- T 3: 10^{10} to 10^{11}
- T 4: 10^{15} to 10^{16}
- T 5: 10^{20} to 10^{21}
- T 6: 10^{25} to 10^{26}

In each test run the Collatz sequence is calculated for all sample numbers. A term by term analysis determines if a term is an HN, that is, an end point of one cMTS and the transition to a new one. The lengths of the Collatz sequence as a whole and the MTS segments are recorded. The HN level at each transition is also determined. Here is a partial but representative sample of the output.

```
Grid[PrependTo[collar, {"Base ", "Term Count ", "MTS Base lengths", "Mean ", "Longest",  
"HN index", "Count ", "Max index"}], Dividers -> {{False}, {False, True, False}}]
```

Base	Term Count	MTS Base lengths	Mean	Longest	HN index	Count	Max index
432715	30	{3, 2, 9, 2, 1, 3, 10}	4.29	10	{3, 3, 2, 3, 2, 3}	6	3
673511	46	{13, 5, 1, 6, 2, 1, 1, 2, 5, 10}	4.6	13	{2, 2, 2, 2, 2, 3, 3, 2, 3}	9	3
566183	26	{9, 5, 3, 1, 3, 5}	4.33	9	{4, 2, 2, 3, 2}	5	4
121755	28	{6, 2, 4, 2, 1, 3, 10}	4.	10	{4, 2, 2, 3, 2, 3}	6	4
514939	69	{8, 5, 5, 2, 3, 7, 26, 4, 7, 2}	6.9	26	{2, 2, 2, 2, 3, 2, 2, 2, 3}	9	3
241419	52	{6, 8, 4, 15, 11, 1, 7}	7.43	15	{3, 2, 2, 2, 3, 2}	6	3
437705	42	{4, 6, 1, 3, 10, 1, 2, 2, 6, 2, 5}	3.82	10	{2, 2, 2, 2, 5, 2, 2, 2, 2, 2}	10	5
613393	55	{3, 7, 11, 1, 20, 4, 7, 2}	6.88	20	{2, 4, 3, 2, 2, 2, 3}	7	4
431663	54	{15, 8, 1, 4, 12, 7, 2, 5}	6.75	15	{2, 2, 2, 2, 3, 2, 2}	7	3
384195	51	{2, 3, 9, 24, 4, 7, 2}	7.29	24	{2, 3, 4, 2, 2, 3}	6	4

The fields are as follows:

- Base: the initial base (term) of the Collatz sequence.
- Term count: the number of bases (terms) in the Collatz sequence.
- MTS base lengths: The successive lengths of each cMTS segment.
- Mean: The mean of the cMTS segment lengths.
- Longest: The longest cMTS segment.
- HN Index: The successive indices of the HN which ends a cMTS segment.
- Count: the counts of HN. This number will always be 1 less than the number of cMTS since an HN ends a cMTS segment, and the last segment ends in 1 (not an HN).
- Max index: The maximum index level of an HN in each Collatz sequence.

This raw data is further analyzed by two report functions. The **rptMTS** function reports the following MTS data:

- Mean base length of the Collatz sequences in the sample.
- Mean number of cMTS per Collatz:
- Mean cMTS segment lengths in all sequences: the lengths of all MTS segments are totaled, and divided by the number of segments.
- Count of all cMTS lengths ≤ 3 : The segment lengths for all MTS segments are pooled, and those ≤ 3 are selected for and tallied. These figures includes lengths of 1.
- Percent: tallied category/total number of segment length.
- Count of all cMTS lengths = 1: Same logic as above.
- Percent:
- Count of all cMTS lengths ≥ 20 : Same logic as above.
- Percent
- Count of Collatz with cMTS lengths ≥ 20 . Different logic: If a Collatz has at least one cMTS segment ≥ 20 , it is tallied, and the percent derived from the total number of sequences. (The coding did not search for or determine if any sequence had more than one 20+ segments.).
- Mean (length) of $20 \geq$ segments.
- Max segment length

The **rptNBR** function pools the total number of neighbors from the HN Index field and breaks out the following categories:

- Total HN2 count and percent
- Total HN3 count and percent
- Total HN4 count and percent
- Total HN5 count and percent
- Total HN6+ count and percent (Everything 6 and over.)
- Max HN index.

Retracing of prior sequences: These studies are focused on the data profile of Collatz sequence.

However, we know from the study of cMTS and the RTS that virtually all Collatz sequence retrace part of the paths of parent sequences. It will necessarily be the case that the data of one sequence will repeat some data from a prior sequence. This cannot be avoided. However, the data in key parameters shows remarkable consistency in all runs suggesting intrinsic dynamics. These factors and others are discussed below.

Findings

Table 1 on next page contains compiled Collatz/CMTS data from all runs.

- Progression. As the sampled numbers increased in magnitude, the Collatz sequences increased in length and the average number of CMTS per Collatz correspondingly increases. This general increase in length is not surprising, and has been found in other empirical studies. Moreover, it is in part a necessary consequence of the retracing of prior sequences.

- Stability with a trend? The average CMTS segment length remains essentially the same, ≈ 5.5 in all ranges. All the values are within one SD except Test 1 at 5.65 (within two SD). I was curious about the slight trend downward so I did four additional tests in sample ranges of 10^{30} to 10^{50} (sample size: 10 K). In each of these additional tests this value was 5.39, so there seems to be a bottom. Note: these four additional studies are not otherwise documented in this file.
- Stability. The percentage of CMTS with a length of 3 or less is highly stable at $\approx 44.5\%$. These short cMTS segments are significant because the tracking Collatz are more likely to fall in value. Note: it is probable that retracing is a factor here and the next two categories. However, in all ranges the probability is surprising stable.
- Stability. The percentage of CMTS with a length of 1 is highly stable at $\approx 17\%$. These CMTS are important because they represent two HN in a row which will drop a tracking Collatz significantly in value.
- Stability. The percentage of CMTS with a length of 20+ is highly stable at $\approx 3.5\%$. The averages fall one standard deviation except for Test at 5% (SD 2). These CMTS are significant because long sequences have a strong tendency to rise. In the cMTS Profile study 99.6% of the sequences rise substantially above their starting number by the 20th term. A tracking Collatz sequence would necessarily fall.
- Progression. The percentage of Collatz sequences that have CMTS 20+ segments increases as the test range increases. This likely reflects the fact that the sequences are longer, and the fact that many sequences inherit long segment from parent sequences (retracing).

Table 1

Category upper limit	Test 1 10 ⁶	Test 2 10 ⁷	Test 3 10 ¹¹	Test 4 10 ¹⁶	Test 5 10 ²¹	Test 6 10 ²⁶
Mean base length of Collatz sequences	46.7	55.6	87.62	127.38	167.61	207.94
Age number of MTS per Collatz Sequence	8.6	9.9	15.99	23.43	30.98	38.53
Mean MTS segment length	5.65	5.59	5.48	5.44	5.41	5.4
Counts percent of MTS segment ≤ 3	36917 45%	44553 45%	72776 46%	107620 46%	142174 46%	177584 46%
Counts percent of MTS segment = 1	13269 16%	16534 17%	27918 17%	41995 18%	55726 18%	70358 18%
Counts percent of MTS segment ≥ 20	3934 5%	4184 4%	5337 3%	6889 3%	8253 3%	9766 3%
Count of Collatz sequence with 20+ segments	3685 37%	3808 38%	4477 45%	5306 53%	5845 58%	6446 64%
Mean length of 20+ segments	25	24.9	24.7	24.64	24.57	24.59
Max segment length	43	53	52	59	60	70

Table 2

Category upper limit	Test 1 10 ⁶	Test 2 10 ⁷	Test 3 10 ¹¹	Test 4 10 ¹⁶	Test 1 10 ²¹	Test 1 10 ²⁶
HN2 count percent	48674 67%	60350 67.44%	100696 67.16%	149816 66.8%	200231 66.8%	250702 66.8%
HN3 count percent	18994 26.17%	22704 25.37%	37909 25.29%	56986 24.41	75728 25.26%	94606 25.21%
HN4 count percent	3598 4.96%	4724 5.28%	8370 5.83%	12958 5.78%	17673 5.9%	22370 5.96%
HN5 count percent	1125 1.55%	1411 1.58%	2354 1.57%	3505 1.53%	4724 1.58%	5808 1.55%
HN6+ count percent	197 0.27%	298 0.33%	595 0.39%	1005 0.45%	1405 0.47%	1789 0.48%
Max HN	10	10	9	10	10	10

Table 2 contains the HN data. Here again the counts increase as the sample range increases, but the percentages remain stable. Essentially 2/3 of the HN are HN2, and another 1/4 are HN3. The remaining 10% is divided in reducing amounts among the higher HN.

Collatz Data

Test 1: Random sample range: 2K to 10⁶, sample size : 10K

Random sample: **Range: 2K to 200K,** **Size: 10K**

```
samp1r= randSamp[2000, 1000000, 10000];
col1r=colProfile[samp1r];//Quiet
```

The CMTS report.

```
rptMTS[col1r]
{Mean base Length of Collatz :           46.69 },
{Avg number of MTS per Collatz:           8.26 },
{Mean MTS Lengths:                       5.65 },
{Count of all MTS lengths ≤ 3:            36917 },
{Percent:                                45% },
{Count of all MTS lengths = 1:            13269 },
{Percent:                                16% },
{Count of all MTS lengths ≥ 20:           3 934 },
{Percent:                                5% },
{Count of Collatz with MTS lengths ≥ 20:   3685 },
{Percent:                                37% },
{Mean of 20+ segments:                    24.98 },
{Max segment length                       43 }
```

The HN report

```
rptNBR[col1r]

{Total HN2:  48674      67.06% },
{Total HN3:  18994      26.17% },
{Total HN4:   3598       4.98% },
{Total HN5:   1125       1.55% },
{HN6+:        197       0.19% },
{Max HN index:           8 }
```

Test 2:

Random sample: **Range: 10⁶ to 10⁷K,** **Size: 10K**

```
samp2r= randSamp[10^6, 10^7, 10000];
col2r=colProfile[samp2r];//Quiet
```

The CMTS report.

```
rptCMTS[col2r]
{Mean base Length of Collatz :           55.64 },
{Avg number of MTS per Collatz:           9.95 },
{Mean MTS Lengths:                       5.59 },
{Count of all MTS lengths ≤ 3:            36917 },
{Percent:                                45% },
{Count of all MTS lengths = 1:            13269 },
{Percent:                                16% },
{Count of all MTS lengths ≥ 20:           3934 },
{Percent:                                5% },
{Count of Collatz with MTS lengths ≥ 20:   3685 },
{Percent:                                37% },
{Mean of 20+ segments:                    24.98 },
```

```
{Max segment length 43 }
```

The HN report

```
rptNBR[col2r]
{Total HN2: 60350 67.44% },
{Total HN3: 22704 25.37% },
{Total HN4: 4724 5.28% },
{Total HN5: 1411 1.57% },
{HN6+: 298 0.33% },
{Max HN index: 10 }
```

Test 3:

Random sample: **Range: 10^{10} to 10^{11} :** **Size: 10K**

```
samp3r= randSamp[10^10, 10^11, 10000];
col3r=colProfile[samp3r];//Quiet
```

The CMTS report.

```
rptCMTS[col3r]
{Mean base Length of Collatz : 87.62 },
{Avg number of MTS per Collatz: 15.99 },
{Mean MTS Lengths: 5.48 },
{Count of all MTS lengths ≤ 3: 72776 },
{Percent: 46% },
{Count of all MTS lengths = 1: 27918 },
{Percent: 17% },
{Count of all MTS lengths ≥ 20: 5337 },
{Percent: 3% },
{Count of Collatz with MTS lengths ≥ 20: 4477 },
{Percent: 45% },
{Mean of 20+ segments: 24.7 },
{Max segment length 52 }
```

The HN report

```
rptNBR[col3r]
{Total HN2: 100696 67.16% },
{Total HN3: 37909 25.29% },
{Total HN4: 8370 5.58% },
{Total HN5: 2354 1.57% },
{HN6+: 595 0.393% }
{Max HN index: 10 }
```

Test 4

Random sample: **Range: 10^{15} to 10^{16} ,** **Size: 10K**

```
samp4r= randSamp[10^15, 10^16, 10000];
col4r=colProfile[samp4r];//Quiet
```

The CMTS report.

```
rptCMTS[col4r]
Mean base Length of Collatz : 127.38 },
{Avg number of MTS per Collatz: 23.43 },
{Mean MTS Lengths: 5.44 },
{Count of all MTS lengths ≤ 3: 107620 },
{Percent: 46% },
{Count of all MTS lengths = 1: 41995 },
{Percent: 18% },
{Count of all MTS lengths ≥ 20: 6889 },
{Percent: 3% },
{Count of Collatz with MTS lengths ≥ 20: 5306 },
{Percent: 53% },
{Mean of 20+ segments: 24.6 },
```

```
{Max segment length 59 }
```

The HN report

```
rptNBR[col4r]
{Total HN2: 149816 67.80% },
{Total HN3: 56876 25.41% },
{Total HN4: 12958 5.79% },
{Total HN5: 3505 1.57% },
{HN6+: 1005 0.45% }
{Max HN index: 10 }
```

Test 5:

Random sample: **Range: 10^{20} to 10^{21} ,** **Size: 10K**

```
samp5r= randSamp[10^20, 10^21, 10000];
col5r=colProfile[samp5r]//Quiet;
```

The CMTS report.

```
rptCMTS[col5r]
Mean base Length of Collatz : 167.61 },
{Avg number of MTS per Collatz: 30.98 },
{Mean MTS Lengths: 5.41 },
{Count of all MTS lengths ≤ 3: 142174 },
{Percent: 46% },
{Count of all MTS lengths = 1: 55726 },
{Percent: 18% },
{Count of all MTS lengths ≥ 20: 8253 },
{Percent: 3% },
{Count of Collatz with MTS lengths ≥ 20: 5845 },
{Percent: 58% },
{Mean of 20+ segments: 24.6 },
{Max segment length 60 }
```

The HN report

```
rptNBR[col5r]
{Total HN2: 149816 67.80% },
{Total HN3: 56876 25.41% },
{Total HN4: 12958 5.79% },
{Total HN5: 3505 1.57% },
{HN6+: 1005 0.45% }
{Max HN index: 10 }
```

Test 6:

Random sample: **Range: 10^{25} to 10^{26} ,** **Size: 10K**

```
samp5r= randSamp[10^20, 10^21, 10000];
col5r=colProfile[samp5r]//Quiet;
```

(see next page)

The CMTS report.

```
rptCMTS[col5r]
Mean base Length of Collatz :      207.94 },
{Avg number of MTS per Collatz:      38.53 },
{Mean MTS Lengths:                    5.41 },
{Count of all MTS lengths ≤ 3:        177584 },
{Percent:                             46% },
{Count of all MTS lengths = 1:        70358 },
{Percent:                             18% },
{Count of all MTS lengths ≥ 20:       9766 },
{Percent:                             3% },
{Count of Collatz with MTS lengths ≥ 20: 6446 },
{Percent:                             64% },
{Mean of 20+ segments:                24.59 },
{Max segment length                   70 }
```

The HN report

```
rptNBR[col5r]
{Total HN2: 250702      66.80% },
{Total HN3: 56876      25.21% },
{Total HN4: 12958       5.96% },
{Total HN5: 3505        1.55% },
{HN6+: 1005            0.48% }
{Max HN index: 10 }
```

Functions

General Functions

See code after the list.

getNBR[b_]: This function calculates the first 10 neighbors of a base.

revSeq[n_, Len_:20]: This function generates the reverse sequence (cMTS) for n. Twenty terms is the default output; the number of terms can be changed with the optional second argument.

colSeq[n_]: This function will calculate the Collatz sequence beginning with n.

randSamp[min_, max_, n_:1000]: This function will generate a random sample of 1000 (default) integers in the range {min, max}. Even numbers are converted to odd number

rnd2[x_]: This function will round a number to two decimal places.

```
getNBR[n_, j_ : 20 ] := Cases[Table[(n*(2^i) - 1)/3, {i, 1, j}], _?IntegerQ]
revSeq[n_, Len_:20]:=Module[{s, p, L, nextN}, p = n; L = {n};
  nextN[e_]:=Which[Mod[e,3]==0, 4e+1, Mod[e,3]==1, (4e-1)/3, Mod[e,3]==2, (2e-1)/3];
  Do[s = nextN[p]; AppendTo[L, s]; p = s, {i, Len*2}]; Take[Select[L, Mod[#,3 ]!=0&], Len]
]
colSeq[x_]:=NestWhileList[If[EvenQ[#1], #1/2, 3 #1+1]&, x, !(#1==4)&, 3]
randSamp[min_, max_, n_:1000]:=If[Mod[#,2]==0, #+1, #]&/@RandomInteger[{min, max}, n ]
rnd2[x_]:=Round[x, .01]
```

Specialized Functions

colProfile

Given a list of odd numbers, this function will profile their Collatz sequences. In calculating these sequences it will determine the length of each cMTS of which that Collatz is composed, and the index of each HN as the Collatz transitions from one cMTS to the next. For each Base in the sample it will report the following data:

Base: Initial base of the Collatz sequence.
Term Count : Count of terms in the sequence.
CMTS lengths: A list of lengths for each cCMTS segment (the base count).
Mean: Mean length of CMTS in that sequence.
Longest: The longest segment.
HN Index: A list of the HN indices of the cBases; the cBases defines each cCMTS.
Count: The count of HN: This should always be one less than the number of cCMTS segments.
Max Index: Highest HN index.

This output is in the form of a list of sublists and data fields. It is not uniquely formatted in order to allow easier querying for specific data. However, several formatting functions are provided below. The only argument is a list of odd numbers. Either **randSamp** (for random numbers) or **contSamp** (for a continuous range) can be used. See general functions above.

```
colProfile[L_List]:=Block[{len, rpt, ind1,nL, b1l, bs,b,nb, bsCnt},ind1=1;len= Length[L];
rpt={};
While[ind1< len+1,
bs=L[[ind1]];nL={}; b= bs;nb= 2;bsCnt = 0; b1l= {};(*We are setting summary
variables;the nb which can be anything but 1 here.*)
While[nb>1,
nb= evalNBR[bs];
If[nb[[2]]==1, bsCnt = bsCnt + 1];
If[nb[[2]]>1, AppendTo[nL, nb[[2]]]; AppendTo[b1l , bsCnt+1];bsCnt = 0];
(*Print[{bs, nb[[1]],b1l, bsCnt, nL}];*)
bs = nb[[1]];nb= nb[[1]];
];
AppendTo[b1l , bsCnt+1]; (*to catch the last bsCnt *)
AppendTo[ rpt, {b,Total[b1l],b1l,rnd2[Mean[b1l]], Max[b1l], nL, Length[nL], Max[nL]}];
nb= 2;bsCnt = 0; b1l= {};nL={}; (*reset variables*)
ind1++;
];
rpt
]
```

evalNBR and nbrLev

The following two function are used by **colProfile**. The function **evalNBR** calculates all terms between a base (neighbor) and the next base. If there are two terms, it is a near neighbor. If there are more than two terms, it calls **nbrLev** to evaluate the neighbor level. This function will get the neighborhood of the new base, and check the position (level) of the prior base in that neighborhood. It will use two protocols to certify the neighbor level. If $N_1 \in S_0$ or $N_1 = 1$, then N_2 is still a near neighbor.

```
evalNBR[b_]:=Module[{nL, nb, ret},ret= {};
nL=NestWhileList[#/2&, 3b+1, EvenQ];
If[Length[nL]==2, ret={Last[nL],1}, ret= nbrLev[b, Last[nL]]];
ret
]
```

```
nbrLev[b1_, b2_]:=Module[{nb, n1},nb= getNBR[b2];
n1=Flatten[Position[nb, b1]];
If[n1[[1]]==2, If[Mod[nb[[1]],3]==0,n1= n1-1]];
If[n1[[1]]==2, If[nb[[1]]==1,n1= n1-1]];
{b2, n1[[1]]} ]
```

Report Functions

The function **rptCMTS** analyzes and reports cMTS -related data from the **colProfile** function. The report categories are:

- **Mean base count of Collatz** : I am counting bases only in each Collatz, and in each cMTS of which they are composed.
- **Avg number of cMTS per Collatz**: The base lengths of each cMTS in each Collatz is recorded in column 3. Each term in this list is thus a separate cMTS, and the average number of terms is calculated.
- **Mean CMTS lengths**: The mean cMTS length for each Collatz is recorded in column 4. This mean is then averaged for the entire set.
- **Count of all cMTS lengths ≤ 3** . The cMTS lengths in column 3 are merged into one list, and the length ≤ 3 are tallied. Note: over these segments a Collatz will always fall in magnitude; i.e. in the segment a, b, c, d , if a, b, c is the 3-segment cMTS, c is HN to d , and $d < a$. Their prevalence is a significant downward pressure.
- **Percent**: The count of 3 or less segments compared to the total lengths in the merged list from above.
- **Count of all cMTS lengths of 1**. This is, again, tallied from the merged list above. Lengths of 1 represent segments that move from one HN immediately to another. They mark a significant drop in magnitude. The **Percent** is also computed.
- **Count of all cMTS lengths ≥ 20** . This is, again, tallied from the merged list above. Lengths of 20 or more have a strong tendency to rise in an cMTS and thus fall in a Collatz that is tracking that cMTS. The **Percent** is also computed.
- **Count of Collatz with cMTS segments ≥ 20** : Column 5 is the longest cMTS segment in each Collatz. Those segments over twenty are tallied and reported here, and their percent of the total number of Collatz is reported in the next column.
- **Mean of the 20+ segments/Max**: This is the mean and max of the above group.

```
rptMts[L_List] := Module[{bsCnt, mtsAL, mtspC, md, lmd, u3, u1, o20, o20s, o20C, tmp},
  bsCnt = rnd2[Mean[L[[All, 2]]]];
  tmp = Flatten[L[[All, 3]]];
  mtspC = rnd2[Total[tmp]/Length[tmp]];
  mtsAL = rnd2[Length[tmp]/Length[L]];
  md = Flatten[L[[All, 3]]]; lmd = Length[md];
  u3 = Length[Select[md, # < 4 &]];
  u1 = Length[Select[md, # < 2 &]];
  o20 = Length[Select[md, # >= 20 &]];
  o20s = Select[md, # >= 20 &];
  o20C = Length[Select[L[[All, 5]], # > 19 &]];
  Grid[{"Mean base Length of Collatz : ",
    ToString[bsCnt]}, {"Avg number of MTS per Collatz: ",
    ToString[mtsAL]}, {"Mean MTS Lengths: ",
    ToString[mtspC]}, {"Count of all MTS lengths \[LessEqual] 3: ",
    ToString[u3]}, {"Percent: ",
    ToString[
      PercentForm[rnd2[u3/lmd]]]}, {"Count of all MTS lengths = 1: ",
    ToString[u1]}, {"Percent: ",
    ToString[
      PercentForm[
        rnd2[u1/lmd]]]}, {"Count of all MTS lengths \[GreaterEqual] 20: \
", ToString[o20]}, {"Percent: ", ToString[PercentForm[rnd2[o20/lmd]]]},
    {"Count of Collatz with MTS lengths \[GreaterEqual] 20: ",
    ToString[o20C]},
    {"Percent: ", ToString[PercentForm[rnd2[o20C/Length[L]]]}],
    {"Mean of 20+ segments: ",
    ToString[rnd2[Mean[o20s]]]}, {"Max segment length ",
    ToString[Max[o20s]]}, Alignment -> {{Right}, {Right}}} ]
```

The function **rptNBR** analyzes and reports HN-related data from the **colProfile** function. Column 6 is a list of HN indices for each Collatz. This data is merged into a single list and the indices are separately tallied. The counts and percents for HN2, HN3, HN4, HN5, and HN6 and above are provided. The max HN index is singled out.

```
rptNBR[L_]:=Module[{nbr, t, u6,mx},
  nbr= ReverseSort[Flatten[L[[A11, 6]]]];
  t= Length[nbr]; nbr=Reverse[Tally[nbr]];
  u6=Total[Select[nbr, #[[1]]<6&][[A11, 2]]];
  mx= Max[L[[A11,8]]];
  Grid[{"Total HN2: ",nbr[[1,2]],Spacer[15], PercentForm[N[nbr[[1,2]]/t]]},{"Total HN3:
",nbr[[2,2]],Spacer[15], PercentForm[N[nbr[[2,2]]/t]]},{"Total HN4:
",nbr[[3,2]],Spacer[15], PercentForm[N[nbr[[3,2]]/t]]},{"Total HN5:
",nbr[[4,2]],Spacer[15], PercentForm[N[nbr[[4,2]]/t]]},{"HN6+: ",t-u6,Spacer[15],
PercentForm[N[(t-u6)/t]]},{"Max HN index: ",Spacer[15], mx}},Alignment->{{Left},
{Right},{Right}}]
```